THE EFFECT OF IONIZATION TRANSPORT ON THE EQUATORIAL F-REGION

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In his discussion of the Appleton ionospheric anomaly[1], Mitra[2] pointed out that ionization produced in the region of the dip equator would diffuse downward along magnetic field lines and hence contribute to the electron concentration at higher latitudes. Rishbeth _et al._[3] and Kendall[4] have shown that diffusion and the usual production and loss processes cannot, however, produce the observed anomalous behavior. It is clear that transport of ionization by other means is also important.

The suggestion has been made[5,6] that vertical electromagnetic drift may be the ionization transport mechanism required. Bramley and Peart[7], using a perturbation approach, have extended the work of Rishbeth _et al._[3] to include the effect of a _small_ electromagnetic drift but a considerable discrepancy with the observations remains.

This letter reports the numberical solution of the steady state continuity equation in the equatorial F-region, incorporating a drift in an upward direction perpendicular to the magnetic field lines and of any magnitude greater than zero. Preliminary results show that it should be possible to develop a quantitative explanation of the Appleton anomaly.

In steady state the continuity equation for the electron number density N is:

$$P - \beta N + D_a D_1 N - D_2 N = 0 \qquad (1)$$

where P is the electron production rate, β is the linear loss coefficient, $D_1 N$ arises from diffusion along the magnetic field lines, D_a is the ambipolar diffusion coefficient, and D_2N is the divergence of the drift term. In solving (1) the natural magnetic dipole coordinates p and q are used, where p = constant defines a particular line of force and q = constant defines an equipotential line. If the Earth's field is taken as a centered dipole, then in terms of the geocentric distance r and the latitude ϕ:

$$\left.\begin{aligned} p &= r/(r_o \cos^2\phi) \\ q &= r_o^{\,2} \sin\phi/r^2 \end{aligned}\right\} \qquad (2)$$

where r_o is a constant with the dimensions of length, taken as the Earth's radius.

The operator D_1 is obtained by modifying that given by Kendall[8] to include the variation with altitude of the scale height of the ionizable constituent, atomic oxygen, and is given by:

$$\begin{aligned} D_1N = {} & \frac{q^2(1+3\sin^2\phi)}{r^2 \sin^2\phi} \frac{\partial N^2}{\partial q^2} - \frac{3q}{Hr} \frac{\partial N}{\partial q} \\ & + \left\{ \frac{2 \sin^2\phi}{H^2(1+3 \sin^2\phi)} + \frac{3 \sin^4\phi+6 \sin^2\phi-1}{Hr(1+3 \sin^2\phi)^2} \right\} N \end{aligned} \qquad (3)$$

where H is the scale height of atomic oxygen. The drift operator which describes the plasma motion perpendicular to the field lines has been derived by Baxter[9]. We have corrected an algebraic sign in Baxter's expression and generalized it to allow for an equatorial (q = 0) drift velocity w_o which varies with altitude, so that, for upward drift:

$$D_2N = \frac{w_o}{r_o}\frac{\partial N}{\partial p} + \frac{1}{r_o}\frac{\partial w_o}{\partial p} N + \qquad\qquad (4)$$

$$+ \frac{4\, w_o\, N}{r(1+3\sin^2\phi)^2}\left\{6\sin^6\phi - 3\sin^4\phi - 4\sin^2\phi + 1\right\}$$

Insertion of (3) and (4) into (1) leads to the parabolic partial differential equation:

$$\frac{\partial N}{\partial p} = a\frac{\partial^2 N}{\partial q^2} + b\frac{\partial N}{\partial q} + cN + d \qquad (5)$$

where the coefficients a, b, c and d are functions of the (p,q) coordinates[10].

The atmosphere is assumed to be isothermal. The Sun is taken to be vertically above the equator so that ϕ is equal to the solar zenith angle and N is symmetrical about the equator. The form adopted for P is the Chapman formula with a correction for the height dependence of gravity. The loss coefficient β is taken proportional to the concentration of the hydrostatically distributed molecular nitrogen. To ease the numerical difficulties D_a, which is assumed to vary inversely with the atomic oxygen concentration at low altitudes, is limited so that it cannot exceed the value taken at some chosen large altitude. This procedure does not affect the results within the accuracy claimed[10].

We integrate (5) by means of an implicit finite-difference method[11], using the boundary conditions (i) $\partial N/\partial q = 0$ at $q = 0$ and (ii) $N = 0$ at the value of q specified by $r = r_o$ on the particular field line being considered. The integration is commenced at the field line corresponding to Z (at q = 0) = 150 km, $Z = r-r_o$ being the height above the Earth's surface, and is continued until a sufficiently large (p,q) space has been covered. The solution after a few integration steps is found to be independent of the initial values adopted for N.

Figure 1 shows contours of N obtained by taking $w_o = 5$ m/sec, and Figure 2 those with $w_o = 15$ m/sec; in both cases w_o is independent of altitude. The values adopted for the other parameters are displayed in Table 1. The results given in the figures are accurate to within less than 5%. In Figure 1 the ratio of the peak value of N_{max} to N_{max} at $\phi = 0$ is 1·6; in Figure 2 this ratio is 3·1. It seems that adjustment of the parameters within the bounds of the present uncertainties in them can produce the main features of the Appleton anomaly. A more complete treatment will be published later[10].

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TABLE 1 Equatorial values of parameters at 400 km

$T(^\circ K)$	$P(cm^{-3}\ sec^{-1})$	$\beta(sec^{-1})$	$D_a(cm^2\ sec^{-1})$	Atomic oxygen concentration (cm^{-3})
1.50×10^{3}	5.00×10^{1}	1.18×10^{-5}	1.27×10^{11}	1.10×10^{8}

CAPTIONS

Figure 1 Contours of constant electron concentration in units of 10^6 cm^{-3} as a function of altitude and magnetic latitude, with w_o = 5 m/sec independent of altitude. Pertinent atmospheric parameters are given in Table 1.

Figure 2 Contours of constant electron concentration in units of 10^6 cm^{-3} as a function of altitude and magnetic latitude, with w_o = 15 m/sec independent of altitude. Pertinent atmospheric parameters are given in Table 1.

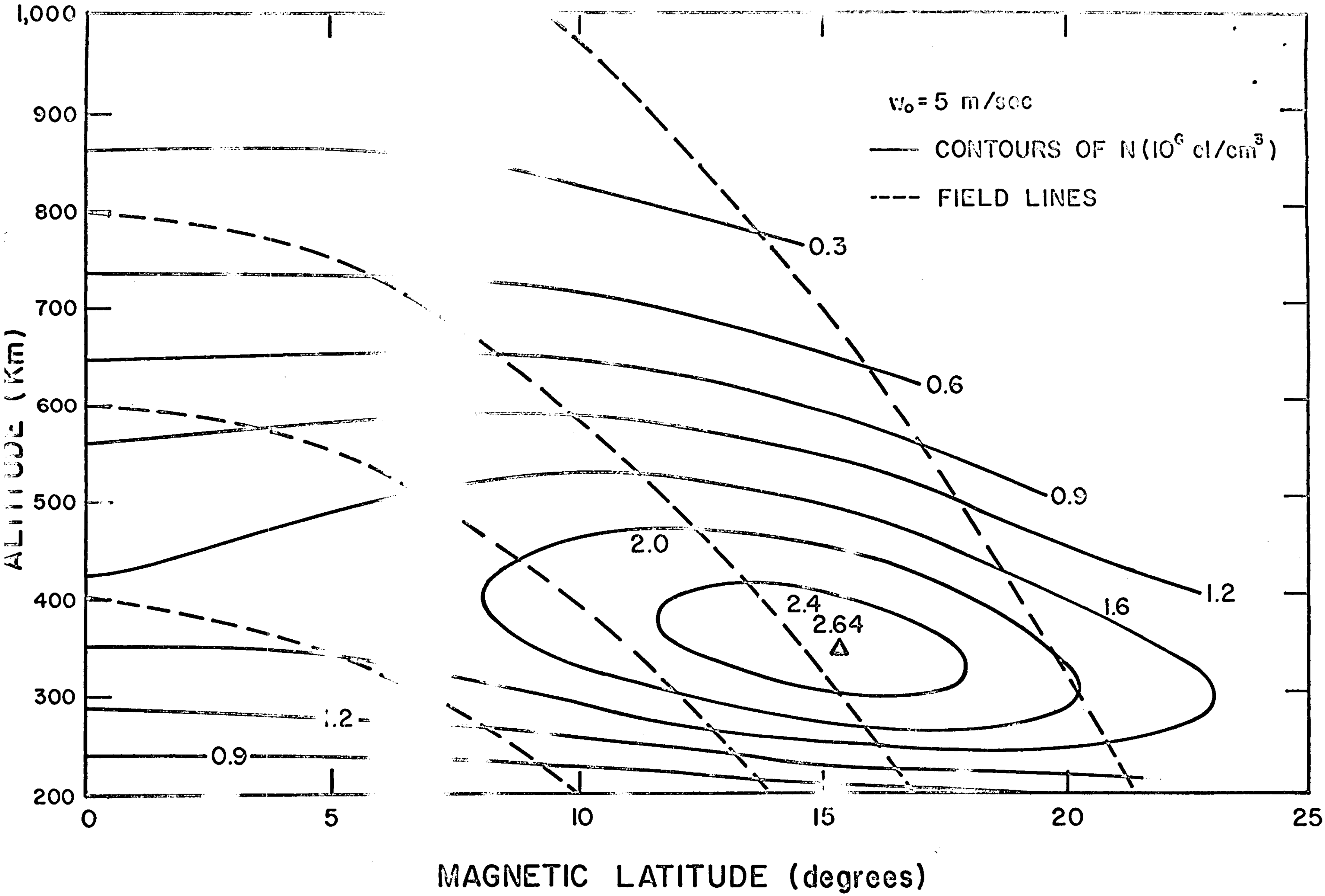
w_o = 5 m/sec
CONTOURS OF N (10^6 el/cm^3)
FIELD LINES
0.3
0.6
0.9
1.2
1.6
2.0
2.4
2.64
1.2
0.9
1,000
900
800
700
600
500
400
300
200
ALTITUDE (Km)
0
5
10
15
20
25
MAGNETIC LATITUDE (degrees)

FIGURE 1

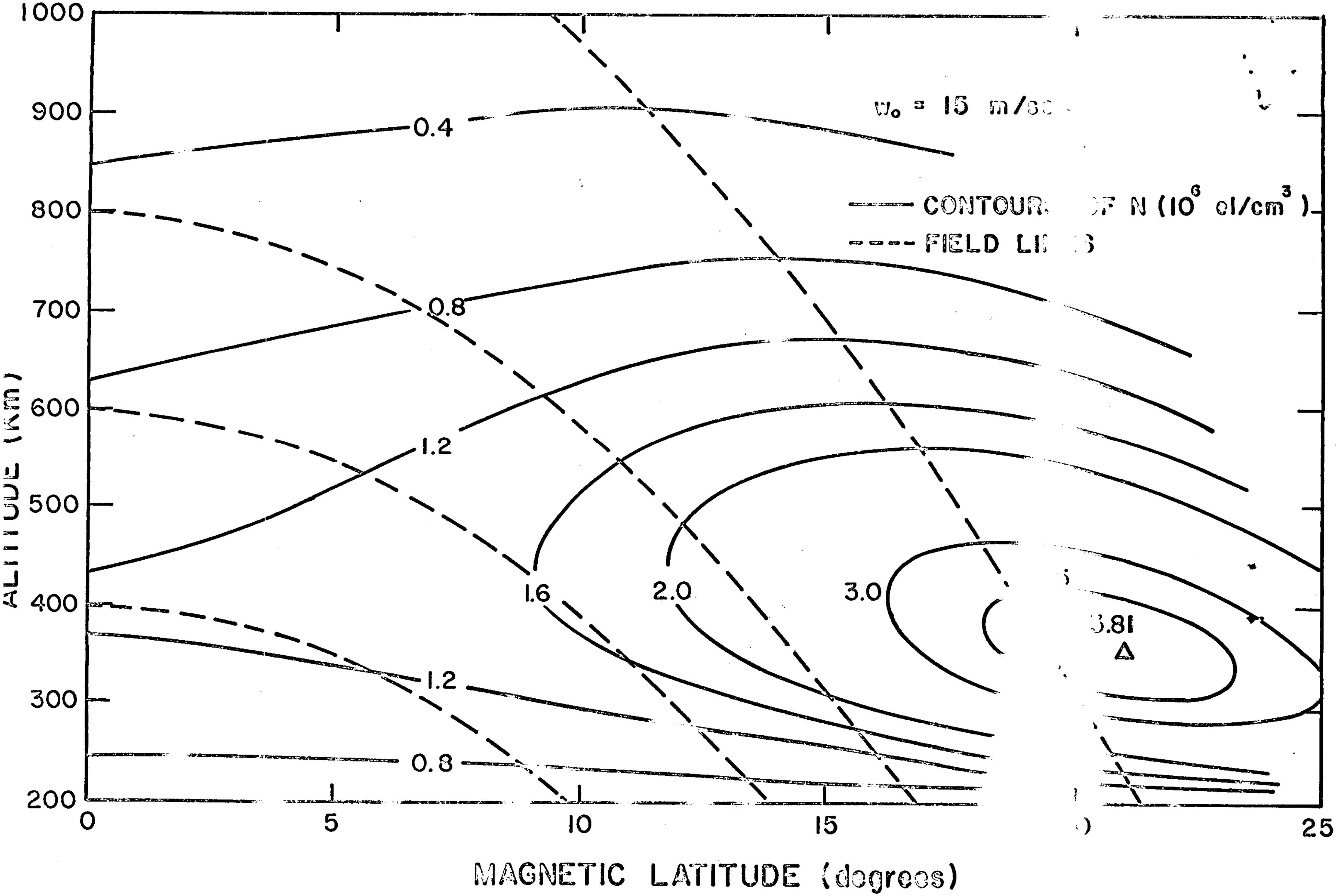
1000
900
800
700
600
500
400
300
200
ALTITUDE (Km)
0
5
10
15
25
MAGNETIC LATITUDE (degrees)
0.4
0.8
1.2
1.6
2.0
3.0
1.2
0.8
3.81
W₀ = 15 m/s
CONTOUR OF N (10⁶ el/cm³)
FIELD LI

FIGURE 2